CONCEPTS OF ALGEBRA

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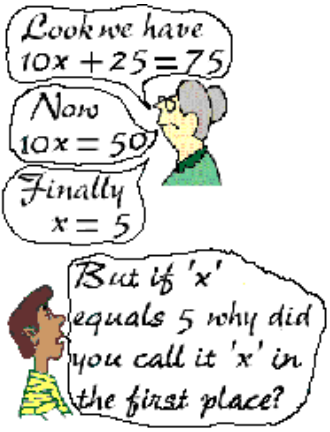
These notes were prepared for students at Macquarie University in Australia but are freely available to anyone. However if you make use of them and are not a Macquarie University student it would be nice if you could email me at christopherdonaldcooper@gmail.com to let me know where you are from. And, if you are from outside of Australia perhaps you could send me a postcard of where you are from to pin up on my wall (Christopher Cooper, 31 Epping Avenue, EASTWOOD, NSW 2122, Australia).

INTRODUCTION

The name ‘algebra’ is derived from the Arabic ‘al-jabr’, meaning ‘restoration’. It builds on arithmetic. Instead of carrying out the arithmetic operations of addition, subtraction, multiplication and division with numbers, we include symbols that *represent* numbers.



The advantage of using symbols is two-fold. Firstly there are many arithmetic facts that are very similar and we can condense infinitely many statements about numbers into a single algebraic one.

For example, instead of saying that $1 + 2 = 2 + 1$ and $2 + 3 = 3 + 2$ and $5 + 8 = 8 + 5$, and so on, we can simply say that $x + y = y + x$ for all numbers x and y. Here the symbols are being used as variables.

The second advantage of using symbols is that we may not know a certain number, but we have information about it from which we can eventually discover the number. This is called “solving an equation” and in this context the symbol is said to represent an ‘unknown’.

Although these notes are called “Elementary Algebra” they are pitched at a level above that which would be appropriate when one was meeting algebra for the first time. The title is designed to distinguish the topics

from the more specialised areas of algebra, such as Linear Algebra and Abstract Algebra.

In chapter 1 we discuss numbers, just ‘real’ numbers at this stage – those that live on the number line. And we discuss the basic algebra of real numbers. Chapter 2 discusses inequalities and absolute values. Then come chapters on induction, finite series and the Binomial theorem. Another couple of chapters deal with the exponential, logarithmic and trigonometric functions that are so useful, both for algebra and for calculus.

At this point we leave the real numbers and its associated algebra, to explore three very important systems: complex numbers, which behave like the real numbers, only better; polynomials and matrices, which requires a whole new look at algebra.

Finally we discuss complex numbers and systems of equations – just two equations in two variables and three equations in three variables.

CONTENTS

7. TRIGONOMETRIC FUNCTIONS

8. LOGS AND EXPONENTIAL FUNCTIONS

9. COMPLEX NUMBERS

10. SYSTEMS OF EQUATIONS